

Generation of Geomagnetic Disturbances in the Ionosphere by a Tsunami Wave

V. M. Sorokin^{a, *}, A. K. Yashchenko^a, and V. V. Surkov^{a, b}

^a*Pushkov Institute of Terrestrial Magnetism, Ionosphere, and Radio Wave Propagation (IZMIRAN),
Russian Academy of Sciences, Troitsk, Moscow oblast, 142190, Russia*

^b*Systems of Precise Instrument Engineering Research and Production Corporation, Moscow, Russia*

*e-mail: sova@izmiran.ru

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Abstract—The mechanism of the generation of the geomagnetic field disturbance accompanying tsunami wave propagation is considered. Electric currents in the marine environment and the ionosphere are the source of the disturbance. The current in the marine environment arises as a result of its motion in the tsunami wave, while the current in the ionosphere occurs due to the occurrence of an acoustic-gravity wave (AGW) propagating from the atmosphere on the ionosphere. The source of the AGW is the vertical displacement of the surface of the marine environment during tsunami-wave propagation in it. Although the ionospheric conductance is significantly smaller than the conductivity of the marine environment, the current value in it may considerably exceed the current value in the marine environment due to the exponential growth in the AGW amplitude during AGW upward propagation. The spatial distribution of a disturbance in the induction of the magnetic field of electric currents flowing in the marine environment and in the ionosphere is obtained with allowance for their mutual inductance. It is shown that the generation of the ionospheric electric current considerably changes the characteristics of the geomagnetic field disturbance induced by a tsunami wave. Calculations have demonstrated the possibility of space monitoring of tsunami waves with the use of satellites to record disturbances of the geomagnetic field.

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1. INTRODUCTION

The motion of the conductive marine environment in a tsunami wave in the geomagnetic field leads to the occurrence of an electric current and, consequently, to geomagnetic disturbances. Toh et al. (2011) give the results of the recording of geomagnetic variations associated with the passage of the tsunami wave from two earthquakes in the region of the Kuril Islands on November 15, 2006, and January 13, 2007. These made it possible to determine the direction of tsunami-wave propagation. The amplitude of the horizontal component of geomagnetic disturbance reversed sign at the stages of wave elevation and damping. The electromagnetic fields generated by motion of the marine environment can be recorded on the land surface and the ocean floor. Data on the recording of the geomagnetic disturbances accompanying the tsunami, which arose as a result of the strong earthquake in Chile on February 27, 2010, are given by Manoj and Maus (2011). Measurements conducted at a distance of 3500 km west of the epicenter detected a periodic signal with an amplitude of 1 nT, which is concurrent with the tsunami-wave onset. Zhang et al. (2014) present a numerical simulation of the magnetic field disturbances that occur due to tidal flows and tsunami waves. The model

is compared with observation results. Wang and Liu (2013) studied magnetic anomalies induced by tsunami waves in the ocean. The performed analysis showed that a tsunami wave 1 m in height is accompanied by a magnetic field disturbance with a 10-nT amplitude on the ocean surface. Its amplitude depends on both the wave parameters and geomagnetic field orientation.

Peltier and Hines (1976) were the first to discuss the possibility of recording a tsunami using methods for monitoring of ionospheric disturbances. The ionosphere is affected by an acoustic-gravity wave (AGW) that propagates upwards in the atmosphere from the ocean surface up to heights of the ionosphere. Artru et al. (2005) obtained an image of the ionospheric disturbance using ionospheric sounding with the GPS-receiver network. It was shown that the ionospheric disturbance is related to the tsunami wave that reached the Japanese coast 22 h after the earthquake in Peru on June 23, 2001. The characteristics of the propagation of the tsunami-generated internal gravity waves in the neutral atmosphere and their interaction with the ionospheric plasma were considered by Occhipinti et al. (2008). The geomagnetic field role in the interaction between the neutral and ionized components of the ionosphere in its equatorial and

midlatitude regions is analyzed. Hickey et al. (2009) performed a numerical simulation of the AGW spectral characteristics and their interaction with the ionospheric F region. The results were used to interpret the tsunami-related disturbances of the electron density in the ionospheric F region and the total electron content. The characteristic velocity of propagation of disturbances and their horizontal spatial scale are 200 m/s and 400 km, respectively. The results of the observation of ionospheric disturbances after earthquakes on Kuril Islands (2006), Samoa Island (2009), and in Chile (2010) were discussed in (Rolland et al., 2010). Data on variations in the total electron content, which were received from the network of GPS receivers located on Hawaiian Islands, were analyzed. The ionospheric disturbances that hypothetically accompany the tsunami-wave displacement of the marine environment level were detected. The disturbances were caused by gravity wave propagation in the ionosphere with the velocity, direction, and time of its arrival, which correlated with the tsunami. Galvan et al. (2012) carried out observations of the ionospheric disturbances arising during the earthquake and tsunami in Tohoku on March 11, 2011. Disturbances of the total electron content were recorded with the GPS-receiver network in Japan. Ionospheric disturbances moving in the direction from the epicenter at velocities of 3400, 1000, and 200–300 m/s are detected. The internal acoustic-gravity waves in the atmosphere, generated by tsunami waves, have been studied by the model and numerical methods (Ma et al., 2015; Wei et al., 2015). The impact of the electric dynamo-field in the ionosphere on the electron density and total electron content in the AGW presence was considered. Coïsson et al. (2015) obtained data on the observation of ionospheric disturbance above the Pacific Ocean, which is probably associated with tsunami near the Tohoku coast. The tsunami arose after the March 11, 2011 earthquake. The data were obtained with the satellite monitoring system. Analysis of the data on the recording of the ionospheric total electron content showed that the characteristic vertical wavelength was 50 km, while the disturbance region covered the height interval on the order of 200 km.

The authors of the works above believe that tsunami-wave propagation is accompanied by AGW generation and by significant ionospheric disturbance. Sorokin and Fedorovich (1982) have shown that the disturbance of the ionosphere state due to ionospheric plasma motion in the geomagnetic field or as a result of a change in its ionization in the zone of flowing of ionospheric currents is accompanied by geomagnetic field variations. Consequently, a tsunami wave generates a geomagnetic field disturbance that is formed not only by the electric current in the marine environment but also by the current in the ionosphere. The electric current arises in the marine environment due to its motion in the tsunami wave in the geomagnetic field. The electric current in the ionosphere arises as a result

of its motion in the AGW in the geomagnetic field. This wave is generated by vertical displacement of the surface of marine environment due to the tsunami wave propagation. The AGW then propagates through the atmosphere upwards, to the ionosphere. The electric current amplitude, both in the marine environment and in the ionosphere, is proportional to the product of the conductivity and the velocity of motion of each of these media. Although the marine environment conductivity exceeds the ionosphere conductance by three to four orders of magnitude, the velocity of the motion of the conductive ionosphere is three to four orders of magnitude higher than the velocity of the motion of the marine environment in the tsunami wave. It is caused by the exponential growth of the gas velocity in the AGW during its upward propagation. As shown below, the value of the current density in the ionosphere is comparable to or higher than the current density in the marine environment. In addition, the interaction of currents due to the mutual inductance of the conductive ionosphere and marine environment is possible. In the work, the aforementioned mechanisms of the formation of the geomagnetic field disturbance are analyzed. A model of a medium with a vertical magnetic field is used to estimate the contribution of each mechanism to the disturbance amplitude. In the context of this model, less cumbersome expressions can be derived; they make it possible to analyze visually each mechanism of generation and to compare its efficiency and also to understand the influence of the medium parameters on signal formation.

2. EQUATIONS OF AN ELECTROMAGNETIC FIELD

We consider a model of the generation of geomagnetic disturbances by a tsunami wave; the scheme of the currents forming in it is given in Fig. 1. The left-hand side of the figure gives the currents in the marine environment and ionosphere, which are connected by means of the longitudinal current to the current in the conjugate ionosphere in the right-hand side of the figure. We introduce the right-handed Cartesian coordinate system with the z axis directed upward vertically. The x axis coincides with the direction of tsunami-wave propagation. We locate the origin of coordinates on the marine environment surface. The geomagnetic field \mathbf{B}_0 is directed upward vertically. The marine environment is located in the layer $-h < z < 0$. Its conductivity σ_w is independent of depth. The maximum of the thin conductive ionospheric layer is located at a height of $z = z_1$; the ionospheric conductance is characterized by the conductivity tensor $\hat{\sigma}$. The magnetosphere, in which conductivities across the magnetic field are zero, is located above the ionosphere. The nonconductive atmosphere is between the marine environment and the ionosphere. The bottom rocks are located in the semispace beneath the marine environ-

ment bed, $z < -h$; the rocks' conductivity is disregarded in comparison to the conductivity of marine environment. The motion of a conductive medium in the geomagnetic field leads to the generation of an electric current with the density \mathbf{j} and to the magnetic field disturbance $\mathbf{b} = \mathbf{B} - \mathbf{B}_0$. The electric field \mathbf{E} and magnetic field disturbance \mathbf{b} are determined from the Maxwell equation system, in which the bias current is ignored in comparison to the conduction current \mathbf{j} :

$$\nabla \times \mathbf{b} = \mu_0 \mathbf{j}; \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{b}}{\partial t}; \quad \nabla \cdot \mathbf{b} = 0, \quad (1)$$

where μ_0 is the permeability, while the conduction-current density \mathbf{j} is determined from the Ohm's law for moving media. Since the magnetic field disturbance is small, $b \ll B$, then the current in the marine environment is defined by Ohm's law

$$\mathbf{j} = \sigma_w (\mathbf{E} + \mathbf{V} \times \mathbf{B}_0), \quad (2)$$

where $\mathbf{V}(\mathbf{r}, t)$ is the fluid velocity field in the tsunami wave. Let us consider an electromagnetic disturbance generated by a tsunami wave propagating in the plane (x, z) . Believing that $\partial/\partial y = 0$, we derive equations for component E_y of the electric field disturbance in the marine environment:

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} - \mu_0 \sigma_w \frac{\partial E_y}{\partial t} = -\mu_0 \sigma_w B_0 \frac{\partial V_x}{\partial t}. \quad (3)$$

The skin-layer value in the marine environment is equal to $\sqrt{\tau/2\pi\mu_0\sigma_w} \approx 10$ km, which considerably exceeds the marine environment depth. The value $\tau \approx 10$ min of the period of the electric field change, which coincided with the tsunami-wave period, was used for estimation, and the value of the marine environment conductivity $\sigma_w = 3$ S/m was chosen. Consequently, the electric field in the marine environment varies only slightly with respect to its depth. Additionally, the horizontal component of the velocity in the tsunami wave is almost constant over the marine environment depth. This makes it possible to integrate Eq. (3) with respect to height under the condition $\sigma_w h = \text{const}$; $h \rightarrow 0$ and to derive the boundary condition for component E_y with transition through the thin conductive layer of the marine environment. As a result we obtain

$$\left\{ \frac{\partial E_y}{\partial z} \right\}_{z=0} - \mu_0 \Sigma_w \frac{\partial E_y}{\partial t} = -\mu_0 \Sigma_w B_0 \frac{\partial V_x(x, 0, t)}{\partial t}; \quad (4)$$

$$\{E_y\}_{z=0} = 0,$$

where $\Sigma_w = \sigma_w h$ is the integral conductivity of the marine environment and braces $\{...\}$ denote the difference in the value of quantities at the upper and lower boundaries of the conductive layer.

We consider the field in the ionosphere generated by an AGW due to its propagation upward from the

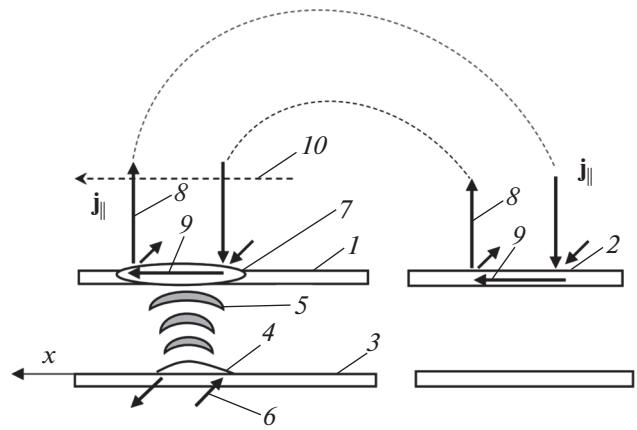


Fig. 1. Scheme of the formation of a system of currents: (1) conductive layer in the lower ionosphere; (2) conductive layer in the conjugate ionosphere; (3) marine environment; (4) tsunami wave; (5) acoustic-gravity wave; (6) electric current in the marine environment; (7) ionospheric disturbance; (8) longitudinal current in the upper ionosphere and magnetosphere; (9) electric current in the ionosphere and in the conjugate ionosphere; and (10) satellite trajectory.

marine environment surface, which is disturbed by the tsunami wave. Ohm's law in the ionosphere is written as

$$\mathbf{j} = \hat{\sigma}(\mathbf{E} + \mathbf{v} \times \mathbf{B}_0); \quad \hat{\sigma} = \begin{pmatrix} \sigma_P & \sigma_H & 0 \\ -\sigma_H & \sigma_P & 0 \\ 0 & 0 & \sigma_{11} \end{pmatrix}, \quad (5)$$

where σ_P, σ_H , and σ_{11} are the conductivities of Pederesen, Hall, and the longitudinal conductivity of the ionospheric plasma and $\mathbf{v}(\mathbf{r}, t)$ is the gas velocity in the AGW propagating from the atmosphere. Since $\sigma_{11} \gg \sigma_{P,H}$, in the ionosphere, then, admitting (5) $\sigma_{11} \rightarrow \infty$, we derive $E_z = 0$. Substituting Ohm's law (5) in the Maxwell equations (1), we obtain the equation of the horizontal component of the electric field in the ionosphere

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} - \mu_0 \left(-\sigma_H \frac{\partial E_x}{\partial t} + \sigma_P \frac{\partial E_y}{\partial t} \right) = -\mu_0 \sigma_P B_0 \frac{\partial v_x}{\partial t}. \quad (6)$$

We believe the conductivities σ_P and σ_H differ from zero in the lower ionosphere layer with a thickness of ~ 30 km, reaching a maximum at a height of ~ 120 km. Taking the values of the ionosphere conductance to be equal to $\sigma_{P,H} \approx 3 \times 10^{-4}$ S/m, we obtain the skin-layer value $\sqrt{\tau/2\pi\mu_0\sigma_{P,H}} \approx 10^3$ km, which considerably exceeds the conductive layer thickness. Consequently, the horizontal field component remains unchanged in the vertical direction inside the conductive layer. This allows Eq. (6) to be replaced by boundary conditions

during the transition through the conductive ionospheric layer by means of its integration over the vertical coordinate. The height dependence of the conductivity can formally be substituted by the δ function at the point of its maximum. We introduce the integral conductivities of Pedersen and Hall by the formulas

$$\sigma_{P,H} = \Sigma_{P,H} \delta(z - z_1); \quad \Sigma_{P,H} = \int \sigma_{P,H}(z) dz.$$

Substituting this equality to Eq. (6) and integrating with respect to z , we derive the boundary condition on the thin conductive layer of the ionosphere

$$\left\{ \frac{\partial E_y}{\partial z} \right\}_{z=z_1} - \mu_0 \frac{\partial}{\partial t} (-\Sigma_H E_x + \Sigma_P E_y) = -\mu_0 B_0 \Sigma_P \frac{\partial}{\partial t} v_x(x, z_1, t); \quad \{E_y\}_{z=z_1} = 0. \quad (7)$$

In Eq. (7), we exclude the component E_x , taking advantage of the equation of continuity of the ionospheric current: $\nabla \cdot \mathbf{j} = 0$. Let us integrate this equality over the depth of the conductive ionospheric layer:

$$\frac{\partial}{\partial x} [\Sigma_P E_x + \Sigma_H E_y - \Sigma_H B_0 v_x(x, z_1, t)] = -[j_z(z_1 + 0) - j_z(z_1 - 0)] = -j_{11}, \quad (8)$$

where $j_{11} = j_z(z_1 + 0)$ is the longitudinal current flowing from the ionosphere upward to the magnetosphere and the current on the lower ionosphere boundary is zero: $j_z(z_1 - 0) = 0$. Due to the high conductivity of the magnetosphere along the magnetic lines of force, the longitudinal current in the magnetosphere flows along magnetic lines of force and is closed on the conductive layer of the conjugate ionosphere, as shown in Fig. 1. In the quasi-static approximation, the electric field in the upper ionosphere and magnetosphere is determined from the second equation of system (1): $\nabla \times (\nabla \times \mathbf{E}) = 0$, the components of which have the form

$$\frac{\partial^2 E_x}{\partial z^2} = 0; \quad \frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} = 0.$$

From these equations it follows that component E_x is transported along the lines of force of the geomagnetic field to the conjugate ionosphere, remaining unchanged, while the component E_y decreases upwards at distances from the ionosphere on the order of the horizontal scale of the tsunami. Since this scale amounts to hundreds of kilometers, component E_y does not reach the conjugate ionosphere. The properties of component E_x in the magnetosphere are determined by the quasi-stationary limit of the directed Alfvén wave, while the E_y component properties are determined by the quasi-stationary limit of the magnetosonic wave. Let us write Eq. (8) in the conjugate ionosphere, to which the longitudinal current is transported without a change:

$$\frac{\partial}{\partial x} \Sigma_P E_x = j_{11}. \quad (9)$$

Summing and subtracting Eqs. (8) and (9), we derive

$$E_x = -\frac{\Sigma_H}{2\Sigma_P} [E_y - v_x(x, z_1, t) B_0]; \quad j_{11} = -\frac{\Sigma_H}{2} \frac{\partial}{\partial x} [E_y - v_x(x, z_1, t) B_0]. \quad (10)$$

Substituting the first formula from (10) to the boundary condition on the ionosphere (7) and assuming a dependence of field components on x and t in the form $\exp(ikx - i\omega t)$, we transform boundary conditions (4) and (7) for the field component E_y in the marine environment and in the ionosphere:

$$\left\{ \frac{dE_y}{dz} \right\}_{z=z_1} + i\omega \mu_0 \Sigma_c E_y(z_1) = i\omega \mu_0 \Sigma_c B_0 v_x(k, z_1, \omega); \quad \{E_y\}_{z=z_1} = 0, \quad \left\{ \frac{dE_y}{dz} \right\}_{z=0} + i\omega \mu_0 \Sigma_w E_y(0) = i\omega \mu_0 \Sigma_w B_0 v_x(k, 0, \omega); \quad \{E_y\}_{z=0} = 0. \quad (11)$$

$$\Sigma_c = \Sigma_P + \Sigma_H^2 / 2\Sigma_P$$

The electric field in the atmosphere and beneath the marine environment, in the earth, is determined by the equations of system (1), in which $\mathbf{j} = 0$. In the upper ionosphere and magnetosphere, the electric field is defined by Eqs. (1) and (5), in which $\mathbf{v} = 0$ and components of the conductivity tensor $\sigma_P = \sigma_H = 0$. For component E_y in three media, we have the equation

$$\frac{d^2 E_y}{dz^2} - k^2 E_y = 0.$$

The solution to this equation is

$$z > z_1 \quad e_y = c_1 \exp(-kz) \\ 0 < z < z_1 \quad e_y = c_2 \exp(kz) + c_3 \exp(-kz) \\ z < 0 \quad e_y = c_4 \exp(kz),$$

where $c_{1..4}$ are the arbitrary constants, which are found by the substitution of solutions into the boundary conditions (11). The components of the magnetic field disturbance are determined from the second equation of system (11):

$$b_x = \frac{i}{\omega} \frac{dE_y}{dz}; \quad b_z = \frac{k}{\omega} E_y.$$

Defining the constants, we derive the expressions for the components of the magnetic field disturbance in the atmospheric layer $0 < z < z_1$ as

$$\begin{aligned}
 & b_x(k, \omega, z) \\
 = & B_0 \left[v_x(k, \omega, z_1) F_x^i(k, \omega, z) - V_x(k, \omega) F_x^w(k, \omega, z) \right], \\
 & b_z(k, \omega, z) \\
 = & -iB_0 \left[v_x(k, \omega, z_1) F_z^i(k, \omega, z) + V_x(k, \omega) F_z^w(k, \omega, z) \right],
 \end{aligned} \quad (12)$$

where $v_x(k, \omega, z_1)$ is the horizontal gas velocity in the AGW at the ionospheric height and $V_x(k, \omega)$ is the horizontal component of the marine environment velocity in the tsunami wave. In the equalities in (12), the following designations are introduced:

$$\begin{aligned}
 F_{x,z}^i(k, \omega, z) &= \frac{u_w e^{kz} \mp i(\omega/k) e^{-kz}}{u_i u_w e^{kz_1} + (\omega/k)^2 e^{-kz_1}}; \\
 F_{x,z}^w(k, \omega, z) &= \frac{u_i e^{k(z_1-z)} \mp i(\omega/k) e^{-k(z_1-z)}}{u_i u_w e^{kz_1} + (\omega/k)^2 e^{-kz_1}}, \\
 u_i &= c_i - i\omega/k; \quad u_w = c_w - i\omega/k; \\
 c_i &= 2/\mu_0 \Sigma_c; \quad c_w = 2/\mu_0 \Sigma_w,
 \end{aligned}$$

where the sign (−) corresponds to the x component, while the sign (+) corresponds to the z component. Assuming in (12) that $\Sigma_c = 0; c_i \rightarrow \infty$, we find the expression for the component of the magnetic field disturbance generated by the tsunami wave without the ionospheric influence:

$$b_x(k, \omega, z) = -B_0 \frac{k \exp(-kz)}{kc_w - i\omega} V_x(k, \omega).$$

This expression coincides with the equality derived by Wang and Liu (2013), in which the mechanism of the generation of magnetic field disturbances by a tsunami wave is discussed.

We consider the geomagnetic field disturbance in the upper ionosphere and magnetosphere, which arises due to tsunami-wave propagation in the marine environment. The occurrence of this disturbance is associated with the generation of lateral currents and the longitudinal current in the ionosphere by means of motion of the neutral component. The mechanism of longitudinal current formation during the motion of a neutral component of the ionosphere was considered by Pokhotelov et al. (1999). The AGW acoustic branch is generated in the model of hydrodynamic disturbance given above. This branch produces a longitudinal current that is proportional to the integral Hall's conductivity of the ionosphere Σ_H . In the height region of $z > z_1$, from the first equation of system (1), we have

$$\mu_0 j_{11}(x, t) = \frac{\partial b_y(x, t)}{\partial x}. \quad (13)$$

As shown below, the inequality is valid in the ionosphere: $E_y \ll E_x$. Consequently, in the formulas in (10), we can disregard the component E_y :

$$\begin{aligned}
 E_x(x, z, t) &\approx \frac{\Sigma_H B_0}{2\Sigma_P} v_x(x, z_1, t); \\
 j_{11}(x, z, t) &\approx \frac{1}{2} \Sigma_H B_0 \frac{\partial v_x(x, z_1, t)}{\partial x}.
 \end{aligned} \quad (14)$$

It should be noted that the formulas in (14) are derived for the specific model considered in this work. In the more general case, the additional contribution to the longitudinal current is made by the AGW gravity branch, which is related to the Pedersen conductivity (Pokhotelov et al., 1999). From Eqs. (13) and (14), we obtain the spatiotemporal distribution of the amplitude of disturbance of the geomagnetic and electric fields in the upper ionosphere and magnetosphere:

$$\begin{aligned}
 b_y(x, t) &= \frac{1}{2} \mu_0 \Sigma_H B_0 v_x(x, z_1, t); \\
 E_x(x, t) &= \frac{1}{\mu_0 \Sigma_P} b_y(x, t).
 \end{aligned} \quad (15)$$

From (15) it follows that the geomagnetic field disturbance and the electric field in the height region of $z > z_1$ are generated mainly by gas motion in the conductive layer of the lower ionosphere.

3. GENERATION OF AN ACOUSTIC-GRAVITY WAVE

We consider AGW generation and propagation in the atmosphere, which is generated by the rise of the water-surface in a tsunami wave propagating in the ocean. The equation determining the gas velocity $\mathbf{v}(\mathbf{r}, t)$ in the AGW propagating in the strato-nonuniform isothermal atmosphere in the linear approximation and in the absence of the wind has the form (Nappo, 2002; Gershman, 1974)

$$\frac{\partial^2 \mathbf{v}}{\partial t^2} = \nabla (c^2 \nabla \cdot \mathbf{v} + \mathbf{g} \cdot \mathbf{v}) + (\gamma - 1) \mathbf{g} (\nabla \cdot \mathbf{v}), \quad (16)$$

where $\gamma = 1.4$ is the adiabatic exponent; $c = \sqrt{\gamma p / \rho} = \text{const}$ is the speed of sound; p, ρ are the atmosphere pressure and density; and $\mathbf{g} = -g \mathbf{e}_z$ is the free-fall acceleration. The equations that determine the vertical and horizontal velocity components for the spatiotemporal dependence of the form $\exp(-i\omega t + ikx)$ follow from (16):

$$\begin{aligned}
 \frac{d^2 v_z}{dz^2} - \frac{1}{H} \frac{dv_z}{dz} + \left[\frac{\omega^2}{c^2} + k^2 \left(\frac{\omega_g^2}{\omega^2} - 1 \right) \right] v_z &= 0; \\
 v_x &= \frac{ik}{k^2 - \omega^2 / c^2} \left(\frac{dv_z}{dz} - \frac{v_z}{\gamma H} \right),
 \end{aligned} \quad (17)$$

where $H = c^2 / \gamma g$ is the height of the homogeneous atmosphere and $\omega_g = \sqrt{(\gamma - 1)g/c}$ is the Brunt–Väisälä frequency. The general solution to Eq. (17) has the form

$$v_z(k, z, \omega) = \exp\left(\frac{z}{2H}\right) [A \exp(iqz) + B \exp(-iqz)]$$

$$q = \sqrt{\frac{\omega^2}{c^2} + k^2 \left(\frac{\omega_g^2}{\omega^2} - 1\right) - \frac{1}{4H^2}} \quad (18)$$

On the marine environment surface, $z = 0$, we have $A + B = v_z(k, 0, \omega)$, where $v_z(k, 0, \omega)$ is the vertical component of the AGW velocity on the marine environment surface. The constants A, B in Eq. (18) are determined from the condition of the positive value of the averaged vertical AGW-energy flux. This condition is considered in the Appendix. According to the results derived in the Appendix, solution (18) of the equations in (17) for the vertical and horizontal components of the AGW velocity is defined by Eq. (A8) of the Appendix:

$$v_z(k, z, \omega) = v_z(k, 0, \omega) \exp\left(iKz + \frac{z}{2H}\right),$$

$$v_x(k, z, \omega) = -iv_z(k, 0, \omega) \frac{k(\Gamma - iK)}{k^2 - \omega^2/c^2} \times \exp\left(iKz + \frac{z}{2H}\right),$$

$$K = -\text{sgn}(\omega)q(k, \omega); \quad \Gamma = (2 - \gamma)/2\gamma H. \quad (19)$$

The vertical velocity component in the AGW on the marine environment surface coincides with the vertical velocity component of the fluid on this surface in the tsunami wave:

$$v_z(k, 0, \omega) = V_z(k, 0, \omega). \quad (20)$$

Let us express the vertical and horizontal components of the velocity in the tsunami wave on the marine environment surface, $V_x(k, 0, \omega)$, $V_z(k, 0, \omega)$, by its displacement $\xi(k, \omega)$. Since $V_z(x, z = 0, t) = \partial\xi(x, t)/\partial t$, then we derive the equality $V_z(k, 0, \omega) = -i\omega\xi(k, \omega)$. To determine the horizontal component of the marine environment velocity in the tsunami wave, we take advantage of the incompressibility condition $\nabla \cdot \mathbf{V} = 0$, which, for the components, has the form $ikV_x(k, z, \omega) = -dV_z(k, z, \omega)/dz$. Pelinovsky (2006) showed that, in the approximation of shallow water, $kh \ll 1$, and small surface displacements, $\xi/h \ll 1$, the horizontal component of the velocity in the tsunami wave remains unchanged over the depth. Consequently, the vertical velocity component depends linearly on depth: $V_z(k, z, \omega) = V_z(k, 0, \omega)(1 + z/h)$. As a result, we obtain the sought relation between velocity components in the tsunami wave and the displacement of the marine environment surface:

$$V_x(k, 0, \omega) = \omega\xi(k, \omega)/kh;$$

$$V_z(k, 0, \omega) = -i\omega\xi(k, \omega). \quad (21)$$

Substituting Eqs. (20) and (21) in (19), we derive the expression for the horizontal component of the gas

velocity in the AGW in the ionosphere at the height $z = z_1$:

$$v_x(k, z_1, \omega) = -\xi(k, \omega) \frac{\omega k(\Gamma - iK)}{k^2 - \omega^2/c^2} \exp\left(\frac{z_1}{2H} + iKz_1\right). \quad (22)$$

4. GEOMAGNETIC DISTURBANCES GENERATED BY A TSUNAMI WAVE

Tsunami waves are long surface gravity waves, $kh \ll 1$, which propagate without dispersion at the velocity $a = \sqrt{gh}$. Let a tsunami wave propagate in the positive direction of the x axis. We choose the spatio-temporal dependence of the vertical displacement of the marine environment surface in the wave $\xi = \xi(x - at)$ in the form

$$\xi(x, t) = \frac{\xi_0}{\cosh^2 \chi(x - at)}; \quad a = \sqrt{gh}.$$

Applying the Fourier transform to this formula, we derive

$$\xi(k, \omega) = 2\pi\xi_0 G(k) \delta(\omega - ka);$$

$$G(k) = \frac{\pi k}{\chi^2 \sinh(\pi k/2\chi)}, \quad (23)$$

where ξ_0 is the maximum vertical displacement of the marine environment surface in the wave.

To obtain the spatiotemporal distribution of the gas velocity in the AGW generated by the tsunami wave, we substitute expression (23) into Eqs. (19)–(21) and apply the inverse Fourier transform. Integrating with respect to ω, k with allowance for a choice of the q sign, we derive

$$v_{x,z}(x, z, t) = \frac{1}{\pi} \text{Re} \int_0^\infty \bar{v}_{x,z}(k, z) \exp[ik(x - at)] dk,$$

$$\bar{v}_x(k, z) = -a\xi_0 G(k) \frac{\Gamma + iq}{1 - h/\gamma H} \exp\left(\frac{z}{2H} - iqz\right), \quad (24)$$

$$\bar{v}_z(k, z) = -ia\xi_0 k G(k) \exp\left(\frac{z}{2H} - iqz\right).$$

Expressions (24) allow the regularities of AGW propagation in the atmosphere to be analyzed. Figure 2 presents the distribution of the vertical component of the gas velocity in the vertical plane (x, z) at the time moment $t = x/a$. The function $W(x, z) = v_x(x, z) \exp(-z/2H)$ is calculated with Eq. (24). From the plots it follows that the AGW propagates upward in the atmosphere above the tsunami-wave region. Its amplitude in the ionosphere reaches a value on the order of 10 m/s.

Let us consider the magnetic field disturbance in the atmosphere. We substitute Eqs. (21)–(23) to (12) and apply the inverse Fourier transform. Then, integrating with respect to ω, k with allowance for the choice of q sign, we derive the spatiotemporal distribu-

tion of components of the magnetic field disturbance in the atmospheric layer from the marine environment surface up to the ionosphere:

$$\begin{aligned}
 b_{x,z}(x, z, t) &= \frac{1}{\pi} \operatorname{Re} \int_0^{\infty} \bar{b}_{x,z}(k, z) \exp[ik(x - at)] dk, \\
 \bar{b}_x(k, z) &= -B_0 \frac{\xi_0}{h} a G(k) \left[\frac{h(\Gamma + iq)}{1 - h/\gamma H} \exp\left(\frac{z_1}{2H} - iqz_1\right) \right. \\
 &\quad \left. \times F_x^i(k, ka, z) + F_x^w(k, ka, z) \right], \\
 \bar{b}_z(k, z) &= -iB_0 \frac{\xi_0}{h} a G(k) \left[-\frac{h(\Gamma + iq)}{1 - h/\gamma H} \exp\left(\frac{z_1}{2H} - iqz_1\right) \right. \\
 &\quad \left. \times F_z^i(k, ka, z) + F_z^w(k, ka, z) \right].
 \end{aligned} \quad (25)$$

It should be noted that the inequality $1 - h/\gamma H > 0$ always takes place in the terrestrial atmosphere and marine environments. This condition implies that the tsunami-wave velocity is smaller than the speed of sound in the atmosphere. The functions $F_{i,w}(k, ka, z)$ and the vertical wavenumber $q(k, ka)$, when entering (25), have the form

$$\begin{aligned}
 F_{x,z}^i(k, ka, z) &= \frac{(c_w - ia)e^{kz} \mp ia e^{-kz}}{(c_w - ia)(c_i - ia)e^{kz_1} + a^2 e^{-kz_1}}; \\
 F_{x,z}^w &= \frac{(c_i - ia)e^{k(z_1 - z)} \mp ia e^{-k(z_1 - z)}}{(c_w - ia)(c_i - ia)e^{kz_1} + a^2 e^{-kz_1}} \\
 q &= \sqrt{\frac{(\gamma - 1)}{\gamma h H} - \frac{1}{4H^2} - \left(1 - \frac{h}{\gamma H}\right) k^2}.
 \end{aligned} \quad (26)$$

Let us perform estimations of quantities in (26). Believing that $\Sigma_c = 10$ S, $h = 10^3$ m, we derive $c_i \approx 2 \times 10^5$ m/s and $a \approx 10^2$ m/s. Consequently, the formulas in (26) can be simplified to take advantage of the inequality $c_i \gg a$:

$$\begin{aligned}
 F_{x,z}^i(k, ka, z) &= \frac{1}{2} \mu_0 \Sigma_c \exp[-k(z_1 - z)]; \\
 F_{x,z}^w &= \frac{\exp(-kz)}{(c_w - ia)}.
 \end{aligned} \quad (27)$$

Substituting (27) into (25), we derive formulas to calculate components of the geomagnetic field disturbance in the form

$$\begin{aligned}
 \bar{b}_x(k, z) &= -B_0 \frac{\xi_0}{h} [\Phi_i(k, z) + \Phi_w(k, z)]; \\
 \bar{b}_z(k, z) &= -iB_0 \frac{\xi_0}{h} [-\Phi_i(k, z) + \Phi_w(k, z)]; \\
 \Phi_i(k, z) &= \frac{ah\mu_0 \Sigma_c \exp(z_1/2H)}{2(1 - h/\gamma H)} \\
 &\quad \times G(k)(\Gamma + iq) \exp[-k(z_1 - z) - iqz_1]; \\
 \Phi_w(k, z) &= G(k) \frac{a \exp(-kz)}{c_w - ia}.
 \end{aligned} \quad (28)$$

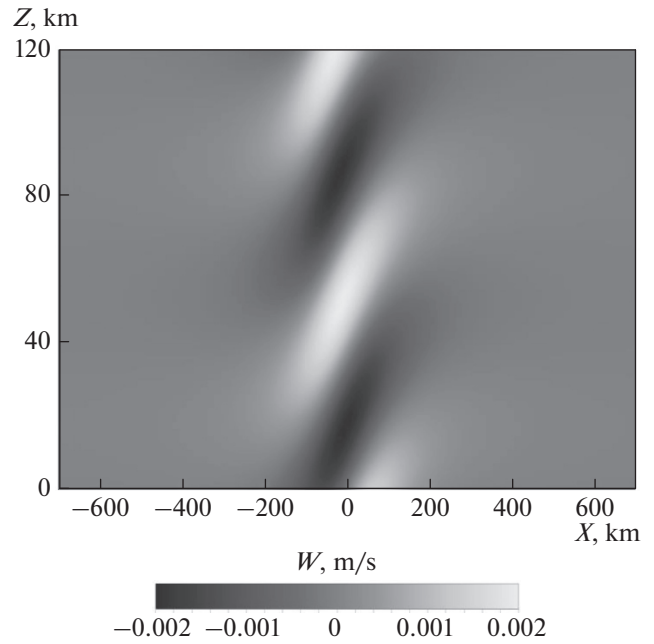


Fig. 2. Calculated spatial distribution of the function $W(x, z)$ in the vertical plane of atmosphere. The parameter values $x_0 = 100$ km and $h = 3$ km are chosen for the calculation.

From (28) it follows that a disturbance of the geomagnetic field generated by a tsunami wave in the atmosphere is composed of two sources. The first source, which is determined by the function $\Phi_i(k, z)$, is the ionospheric current, which arises when the AGW propagates in the ionosphere, while the second source, which is determined by the function $\Phi_w(k, z)$, is the current that arises when the marine environment moves in the tsunami wave. Let us compare the current values. To estimate the ratio of current values in the ionosphere and the marine environment, we choose the function $\Phi_i(k, z)$ at the point $z = z_1$, while the function $\Phi_w(k, z)$ is chosen at the point $z = 0$. Believing that $k \ll 1/\sqrt{hH}$, we obtain from (28)

$$\frac{|\Phi_i|}{|\Phi_w|} \sim \sqrt{\frac{h}{H}} \frac{\Sigma_c}{\Sigma_w} \exp(z_1/2H).$$

Assuming $\Sigma_p = \Sigma_H = 10$ S, $\sigma_w = 4$ S/m, $h = 1$ km, $\Sigma_w = 4 \times 10^3$ S, $z_1 = 120$ km, $H = 8$ km, and $\Sigma_c = 15$ S, we derive

$$\sqrt{\frac{h}{H}} \frac{\Sigma_c}{\Sigma_w} \exp(z_1/2H) \sim 3.$$

Consequently, the amplitudes of the geomagnetic field disturbances, which are generated by the motion of the marine environment in the tsunami wave and the motion of the ionosphere in the AGW, are comparable in magnitude. It should be noted that the value of the integral conductivity under daylight conditions is chosen for estimation. In the nighttime ionosphere,

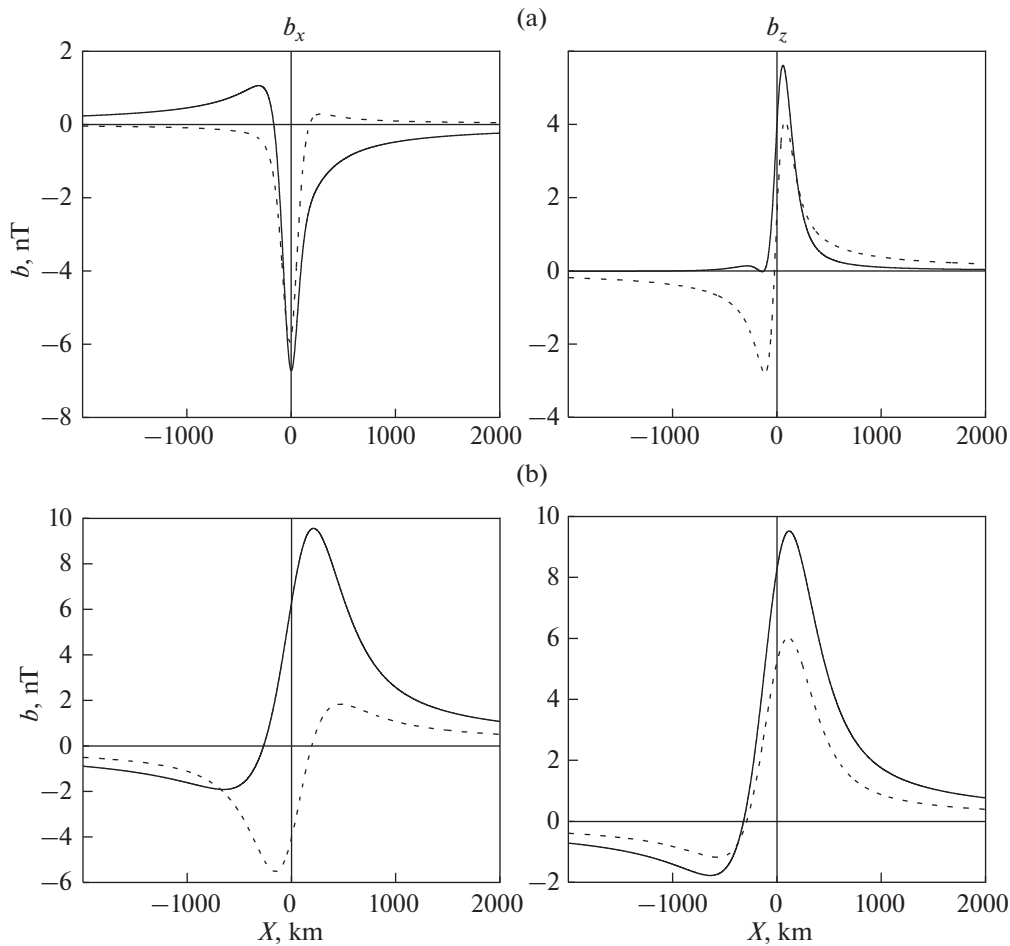


Fig. 3. Calculated horizontal distribution of components of the geomagnetic field disturbance on the marine environment surface. The magnetic field disturbances generated by the electric field in the marine environment are denoted by the dashed line. The solid line designates the magnetic field disturbances generated by electric currents in the ionosphere and in the marine environment. The following parameter values are chosen for calculations: (a) $x_0 = 100$ km, $h = 1$ km; (b) $x_0 = 300$ km, $h = 3$ km.

the integral conductivity is an order of magnitude smaller and, consequently, the contribution from the current flowing in the ionosphere to the observed magnetic field disturbance decreases. Assuming $\Sigma_c = 0$ in (28), we derive the spatiotemporal distribution of the magnetic field disturbance over the marine environment surface without the ionospheric influence:

$$\begin{aligned}
 b_x(x, z, t) &= -B_0 \frac{\xi_0}{\pi h} \operatorname{Re} \left\{ \frac{a}{c_w - ia} \right. \\
 &\times \left. \int_0^{\infty} G(k) \exp[ik(x - at) - kz] dk \right\}; \\
 b_z(x, z, t) &= -B_0 \frac{\xi_0}{\pi h} \operatorname{Re} \left\{ \frac{ia}{c_w - ia} \right. \\
 &\times \left. \int_0^{\infty} G(k) \exp[ik(x - at) - kz] dk \right\}.
 \end{aligned} \quad (29)$$

The amplitudes of disturbances of the geomagnetic field generated by currents in the ionosphere and the

marine environment are calculated with the formulas from (25). For comparison, the amplitude of a disturbance of the geomagnetic field generated by a current in the marine environment without the ionospheric influence are calculated with Eq. (29). The following parameter values are chosen: $B_0 = 5 \times 10^{-5}$ T, $\sigma_w = 4$ S/m, the maximum amplitude of the tsunami wave $\xi_0 = 0.5$ m, the speed of sound in the atmosphere $c = 330$ m/s, the integral conductivities of the ionosphere $\Sigma_p = \Sigma_H = 10$ S, the maximum height of the conductive layer of the ionosphere $z_1 = 120$ km, and the height of the homogeneous atmosphere $H = 8$ km. Figure 3 shows the spatial distribution of components of the geomagnetic field disturbance on the marine environment surface for its different depths and various horizontal scales of the tsunami wave. From plots it follows that the occurrence of the current in the ionosphere keeps the nature of the temporal dependence of the amplitudes of components of the geomagnetic field disturbance over the marine environment sur-

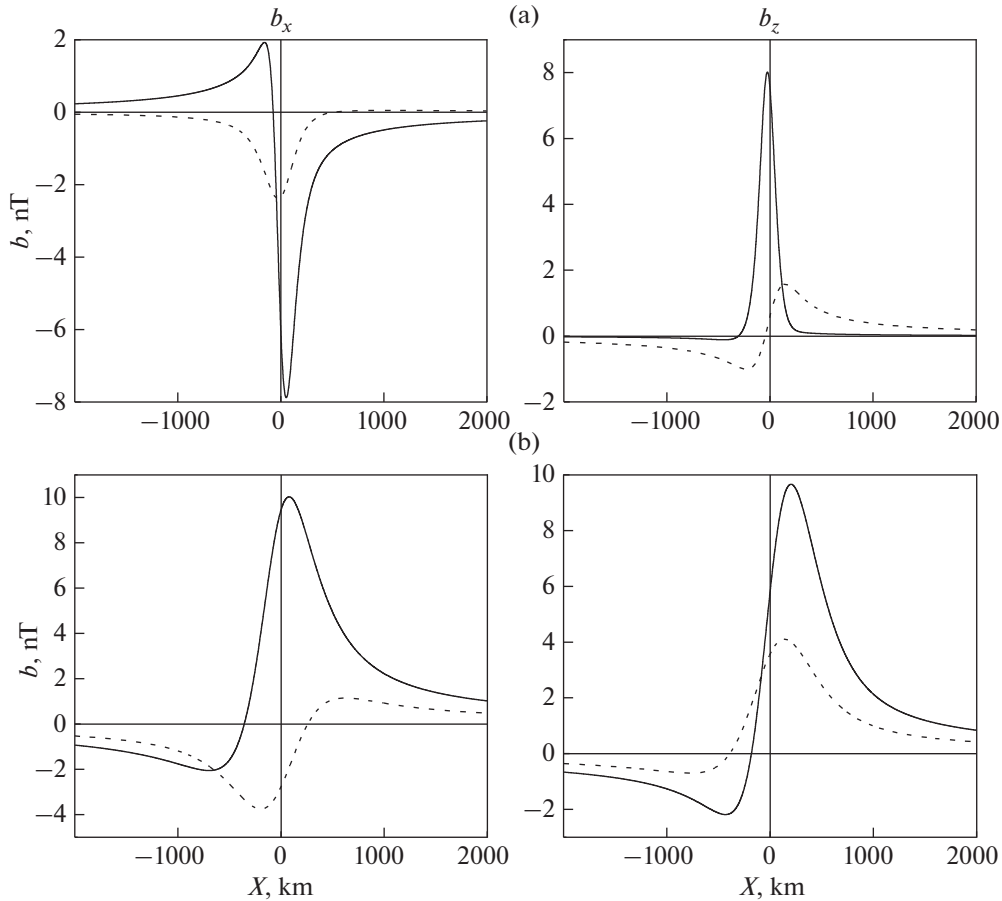


Fig. 4. Calculated horizontal distribution of components of the geomagnetic field disturbance in the ionosphere at a height of 120 km. The dashed line denotes the magnetic field disturbances generated by an electric current in the marine environment. The magnetic field disturbances generated by electric currents in the ionosphere and the marine environment are designated by the solid line. The following values of parameters are chosen for calculations: (a) $x_0 = 100$ km, $h = 1$ km; (b) $x_0 = 300$ km, $h = 3$ km.

face. However, with growth in the horizontal scale of tsunami wave and the marine environment depth, the ionospheric current increases the disturbance amplitude by five times. Figure 4 demonstrates the spatial distribution of components of the geomagnetic field disturbance in the ionosphere at a conductivity-maximum height of 120 km for different depths of the marine environment and various horizontal scales of the tsunami wave. From the plots it follows that allowance for the ionospheric current leads to an increase in the signal amplitude by a factor of 4–5. With an increase in the horizontal scale and the depth of marine environment, the nature of the temporal dependence of the disturbance amplitude is maintained. However, with a decrease in these parameters, a qualitative change in temporal characteristics of signals occurs. Namely, allowance for the ionospheric current results in a sign reversal of the horizontal component of the geomagnetic field disturbance in comparison to the unipolar temporal dependence, whereas the vertical component of the disturbance becomes unipolar. Figure 5 presents the spatial distribution of the component of the geomagnetic field disturbance

generated by the tsunami wave in the vertical plane of the atmosphere. From the figure it follows that allowance for the ionospheric current leads to a fundamental change in the height dependence of the amplitude of a disturbance in the Earth–ionosphere layer. Its value reaches 1–10 nT and does not decrease with height.

In the upper ionosphere and magnetosphere, the geomagnetic field component b_y , which is generated by the AGW according to Eq. (15), differs from zero. Substituting (24) to (15), we derive

$$\begin{aligned}
 b_y(x, t) &= \frac{1}{\pi} \operatorname{Re} \int_0^{\infty} \bar{b}_y(k) \exp[ik(x - at)] dk; \\
 E_x(x, t) &= \frac{1}{\mu_0 \Sigma_P} b_y(x, t); \quad j_{11}(x, t) = \frac{1}{\mu_0} \frac{\partial b_y(x, t)}{\partial x}; \\
 \bar{b}_y(k) &= -\frac{1}{2} B_0 \mu_0 \Sigma_H a \xi_0 G(k) \\
 &\times \frac{\Gamma + iq}{1 - h/\gamma H} \exp\left(\frac{z_1}{2H} - iqz_1\right).
 \end{aligned} \tag{30}$$

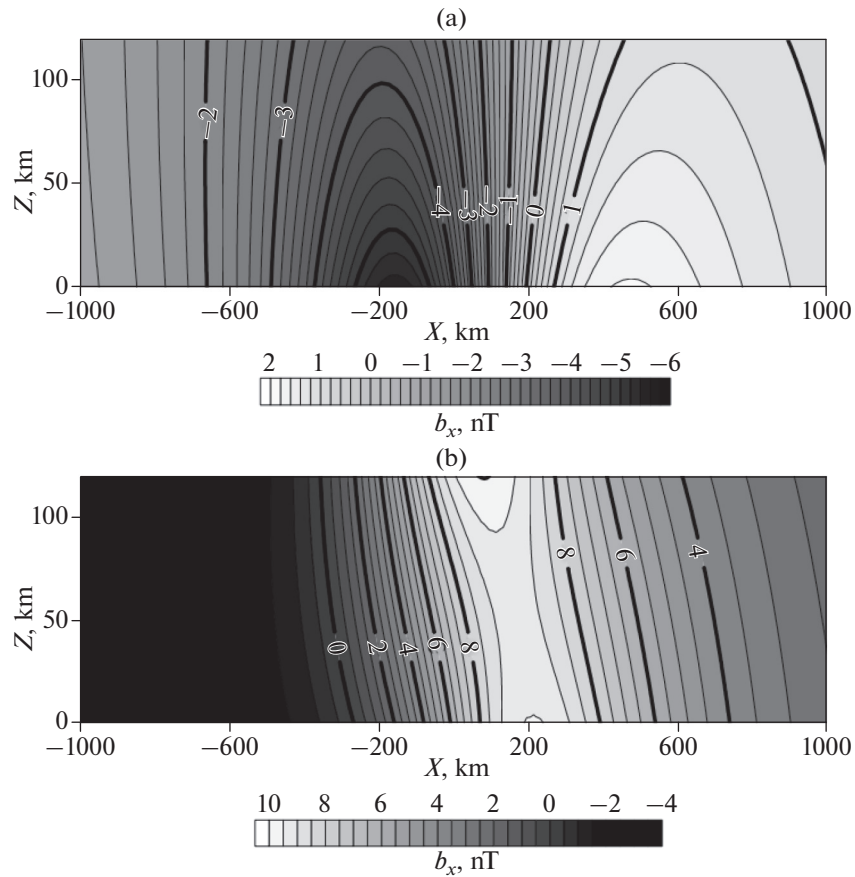


Fig. 5. Calculated spatial distribution of the component b_x of the geomagnetic disturbance in the vertical plane of the Earth-ionosphere layer: (a) geomagnetic disturbance generated by an electric current in the marine environment; (b) geomagnetic disturbance generated by electric currents in the ionosphere and in the marine environment. The following parameter values are chosen: $x_0 = 300$ km, $h = 3$ km.

The formulas in (30) make it possible to calculate the spatiotemporal distribution of the geomagnetic field disturbance, the electric field, and the longitudinal current in the upper ionosphere and magnetosphere. Figure 6 presents results of calculation of the geomagnetic field disturbance and the longitudinal current. From the plots it follows that an increase in the marine environment depth leads to an increase in the horizontal scale of the longitudinal current and its magnetic field. The magnetic field amplitude can reach values on the order of 10 nT, while the longitudinal current amplitude reaches a value of $j_{11} \approx 8 \times 10^{-8}$ A/m².

Let us estimate the value of the electric field components in the region of $z > z_1$. From the equation $\nabla \times \mathbf{E} = -\partial \mathbf{b} / \partial t$ and the condition $\partial / \partial t = -a \partial / \partial x$, we derive $E_y = a b_z$. From the plots it follows that $b_z \sim 1 - 10$ nT at heights of the ionosphere. Assuming $a = 100$ m/s, we obtain $E_y \approx 0.1 - 1.0$ μ V/m. From the plots in Fig. 6, it follows that $b_y \approx 10$ nT. Assuming

$\Sigma_p \approx 1 - 10$ S, from (30) we derive $E_x \approx 1 - 10$ mV/m. Consequently, in (10), we can disregard the quantity $E_y \ll E_x$. From (14) it follows that $E_y \approx 1$ μ V/m, $E_x \approx 10$ mV/m, $j_{11} \approx 10^{-8}$ A/m². It should be noted that the value of component E_x is derived from the electrostatic approximation, while E_y is generated by the induction effects and vanishes in the electrostatic approximation ($E_y = -\partial \phi / \partial y \equiv 0$). The given estimations show that component b_y of the magnetic field of the longitudinal current and component E_x of the electric field in the upper ionosphere and magnetosphere, which are generated by tsunami-wave motion, are totally measurable by satellite methods.

5. CONCLUSIONS

When data on the recording of disturbances of the geomagnetic field generated by the tsunami wave are analyzed, it is believed, as a rule, that an electric current flowing in the marine environment due to its

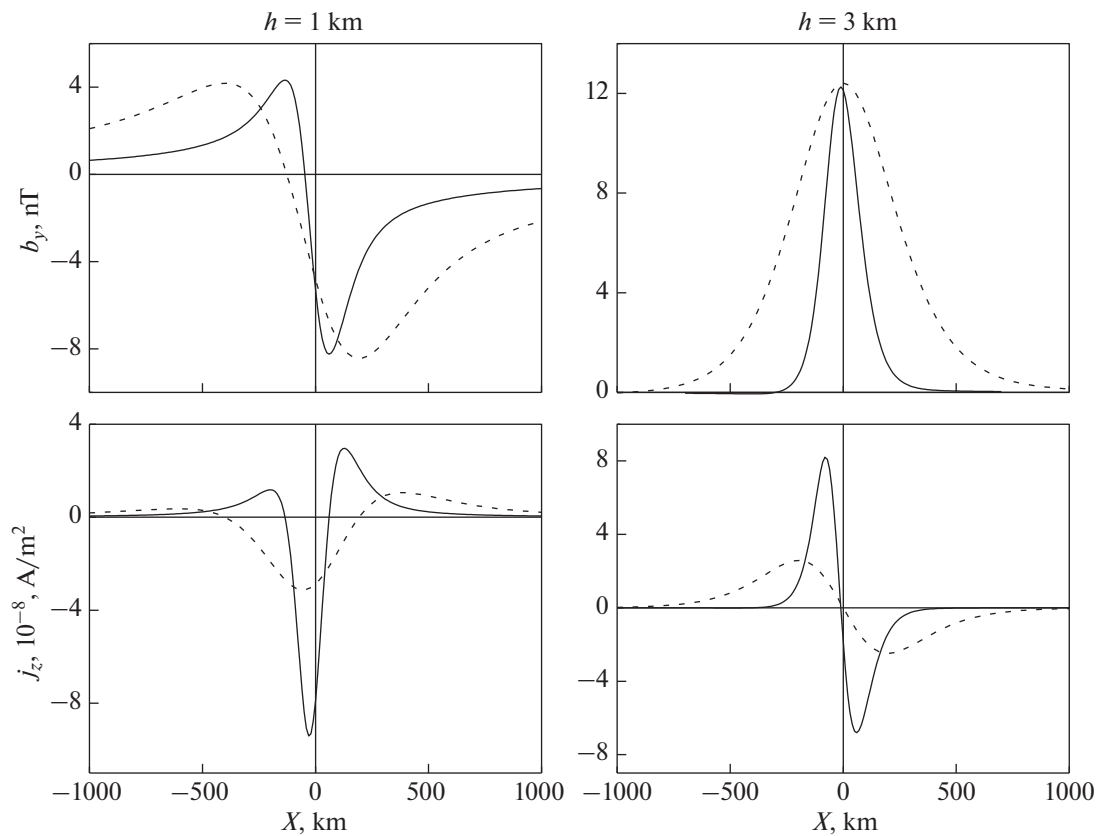


Fig. 6. Calculated amplitudes of the component b_y of geomagnetic field disturbance and the longitudinal current in the upper ionosphere and magnetosphere as functions of the coordinate x with a marine environment depth of: (left panel) 1 km; (right panel) 3 km. The following parameters are chosen for calculations: $x_0 = 100$ km (solid lines); $x_0 = 300$ km (dashed lines).

motion in a tsunami wave serves a source of these disturbances. In this work, it is shown that, in addition to this current, the ionospheric electric current is also a source of the geomagnetic field disturbance. The ionospheric current arises due to the occurrence of the AGW propagating from the atmosphere on the ionosphere. The source of the wave is a vertical displacement of the surface of marine environment during tsunami-wave propagation in it. Although the ionospheric conductance is considerably smaller than the conductivity of the marine environment, the value of the ionospheric current may exceed the current in the marine environment due to exponential growth in the AGW amplitude during its upward propagation. The maximum amplitude of the geomagnetic field disturbance reaches values of 1–10 nT. This estimate is obtained for the daylight ionosphere. In the nighttime ionosphere, the integral conductivity decreases by an order of magnitude, which leads to a reduction of the contribution to the observed signal of the magnetic field of the current generated in the ionosphere as a result of the occurrence of the AGW on the ionosphere.

The presence of an electric current in the ionosphere considerably changes the disturbance charac-

teristics of the geomagnetic field generated by the tsunami wave. The appearance of a current in the ionosphere keeps the nature of the temporal dependence of amplitudes of components of the geomagnetic field disturbance over the marine environment surface. With an increase in the horizontal scale of the tsunami wave and in the marine environment depth, there is a five-fold increase in the disturbance amplitude by the ionospheric current. Allowance for the ionospheric current leads to an increase in the signal amplitude by a factor of 4–5 at ionospheric heights. With an increase in the horizontal scale and the depth of marine environment, the nature of the temporal dependence of the disturbance amplitude remains unchanged. However, with a decrease in these parameters, there is a qualitative variation in the time response of the signals. Namely, allowance for the ionospheric current leads to sign reversal in the horizontal component of the geomagnetic field disturbance in comparison with the unipolar temporal dependence, whereas the vertical disturbance component becomes unipolar. In addition, the height dependence of the disturbance amplitude in the Earth–ionosphere layer is changed. Its magnitude reaches a value of 1–10 nT and does not decrease with height.

In this work, it was shown that the tsunami-generated ionospheric electric current is closed on the conjugate ionosphere by means of longitudinal currents. In this case, lateral components of the magnetic and electric fields arise in the upper ionosphere and magnetosphere. The amplitude of the magnetic field and longitudinal current may reach values on the order of 10 nT and 10^{-8} A/m², while the electric field amplitude reaches a value of 10 mV/m. An increase in the marine environment depth leads to an increase in the horizontal scale of the longitudinal current and its magnetic field. The values of amplitudes of the disturbance of the magnetic and electric fields in the upper ionosphere and magnetosphere, which are generated by the tsunami-wave motion, are easily measurable by satellite methods. This allows a conclusion to be drawn on the possible monitoring a tsunami wave by space methods.

APPENDIX

We consider an atmospheric AGW generated by the uplift of the surface of the marine environment. Let the horizontal and vertical components of the velocity $\mathbf{v} = (v_x, 0, v_z)$ in the AGW and also the pressure disturbance p depend on x, t according to the harmonic law:

$$\begin{aligned} v_x(x, z, t) &= U(z) \exp(-i\omega t + ikx), \\ v_z(x, z, t) &= W(z) \exp(-i\omega t + ikx), \\ p &= P(z) \exp(-i\omega t + ikx). \end{aligned} \quad (\text{A1})$$

The physically meaningful quantities are determined by the real parts of complex expressions (A1). The wave-energy flux is defined by the formula $\mathbf{S} = \text{Re}(p) \text{Re}(\mathbf{v})$ (Yeh and Liu, 1974). Since the energy of the wave propagates from its source, then the \mathbf{S} vertical component, averaged over x, t , should be positive:

$$S_z = \langle \text{Re}(p) \text{Re}(v_z) \rangle_{x,t} = \frac{1}{2} \text{Re}(PW^*) > 0. \quad (\text{A2})$$

The pressure disturbance in the wave is found from the equation of motion

$$\rho_0 \frac{\partial v_x}{\partial t} = -\frac{\partial p}{\partial x}.$$

From this equation, we derive the equality

$$P = \frac{\rho_0 \omega}{k} U; \quad \rho_0(z) = \bar{\rho} \exp(-z/H), \quad (\text{A3})$$

where $\bar{\rho}$ is the sea-level atmospheric density. Substituting (A3) into (A2), we obtain the dependence of the vertical component of the wave-energy flux density on velocity components:

$$S_z = \frac{\omega \bar{\rho}}{2k} \text{Re}(UW^*) \exp(-z/H). \quad (\text{A4})$$

According to Eqs. (17) and (18), the height dependence of velocity components in the wave is written as

$$\begin{aligned} W &= e^{z/2H} (Ae^{iqz} + Be^{-iqz}); \\ U &= e^{z/2H} (\alpha A e^{iqz} - \alpha^* B e^{-iqz}) \\ \alpha &= -ik(\Gamma - iq) / (k^2 - \omega^2/c^2). \end{aligned} \quad (\text{A5})$$

Substituting (A5) to (A4), we derive

$$\begin{aligned} S_z &= \frac{\bar{\rho} \omega}{2k} \text{Re}[\alpha |A|^2 - \alpha^* |B|^2] \\ &= -\frac{\bar{\rho} \omega q}{2(k^2 - \omega^2/c^2)} (|A|^2 - |B|^2). \end{aligned} \quad (\text{A6})$$

The AGW source is the tsunami-induced displacement of the marine environment surface, which moves at the phase velocity $\omega/k = a$ along the x axis. Substituting $\omega = ka$ into the denominator of expression (A6), we derive $1 - \omega^2/(c^2 k^2) = 1 - a^2/c^2 > 0$. Consequently, expression (A6) can be simplified:

$$S_z = -\frac{\bar{\rho} \omega q}{2k^2(1 - a^2/c^2)} (|A|^2 - |B|^2). \quad (\text{A7})$$

Equality (A7) allows us to obtain the condition under which the vertical component of the energy flux density is positive, $S_z > 0$:

$$\begin{aligned} A &= v_z(k, 0, \omega); \quad B = 0 \text{ with } \text{sgn}(\omega) < 0 \\ A &= 0; \quad B = v_z(k, 0, \omega) \text{ with } \text{sgn}(\omega) > 0. \end{aligned}$$

From here it follows that the radiation condition is determined only by the sign of ω and that the solution (18) to Eq. (17) takes the form

$$\begin{aligned} v_z(k, z, \omega) &= v_z(k, 0, \omega) \exp\left(iKz + \frac{z}{2H}\right); \\ K &= -\text{sgn}(\omega) q(k, \omega). \end{aligned} \quad (\text{A8})$$

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Translated by M. Samokhina

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