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SPACE PLASMA

A Possible Interpretation of the Zebra Pattern in Solar Radiation

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Abstract—The nature of the zebra pattern in continual type-IV solar radio bursts is discussed. It is shown that, when a weakly relativistic monoenergetic proton beam propagates in a highly nonisothermal plasma, the energy of the slow beam mode can be negative and explosive instability can develop due to the interaction of the slow and fast beam modes with ion sound. Due to weak spatial dispersion, ion sound generation is accompanied by cascade merging, which leads to stabilization of explosive instability. The zebra pattern forms due to the scattering of fast protons by ion sound harmonics. The efficiency of the new mechanism is compared with that of previously discussed mechanisms.

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1. INTRODUCTION

Stripes in the radiation and absorption spectra in the form of more or less regular harmonics (the so-called “zebra pattern”) against the background of continual radiation of type-IV radio bursts have been studied for nearly half a century since the first publication by Elgarøy [1]. The main properties of the zebra pattern are described in reviews [2–4]. The interpretation of the zebra pattern has always lagged behind the accumulation of new observational data. A dozen models of the zebra pattern have been proposed, the most developed of which are based on the double plasma resonance mechanism [5, 6] and interaction of plasma waves with whistlers [7, 8]. However, these models encounter some difficulties that stimulate the search for new mechanisms. In recent years, more advantageous models based on the propagation of electromagnetic waves through regular density inhomogeneities in the solar corona have been developed [9–11].

Observations of the zebra pattern during solar bursts indicate that particles in the radio source are accelerated to relativistic velocities and different wave modes are excited. Therefore, other possible mechanisms of wave–particle interaction should also be taken into account. For example, decay of whistlers into weakly dispersive, weakly damped ion sound harmonics at frequencies much lower than the ion Langmuir frequency ω_{0i} was considered in [12]. In the present paper, we discuss an alternative mechanism related to the development of explosive instability of a weakly relativistic beam propagating in a nonisothermal plasma. Remind that explosive instabilities develop in nonequilibrium systems in which modes with negative energies exist [13]. In this case, in the resonance triplet, the wave with the highest frequency (ω_3) should have a negative energy, while the waves with the lower frequencies ($\omega_{1,2}$) should have positive energies (see, e.g., [14–16]); or, vice versa, the waves $\omega_{1,2}$ should have negative energies, while the wave ω_3 should have a positive energy. When the wave ω_2 has a negative energy and the waves $\omega_{1,3}$ have positive energies, instability accompanied by the excitation of the highest frequency wave (up-conversion) develops. In this case, the energy of the low-frequency wave ω_1 is transferred to the higher frequency waves $\omega_{2,3}$, the energies of which grow by the exponential law in the given field of the wave ω_1 [17], as is in a conventional decay instability [18]. As will be shown below, the more efficient (from the standpoint of energy transfer) mechanism is generation of “saw-tooth” ion sound due to the development of explosive instability of a weakly relativistic proton beam propagating in a highly nonisothermal plasma. Since the plasma electron temperature T_e is much higher than the ion temperature, Landau damping of ion sound can be ignored [19] and ion motion can be described in terms of quasi-hydrodynamic equations [19]. However, viscous damping of ion sound should be taken into account, because it is proportional to the sound wavenumber squared and the viscosity coefficient is $\eta \approx \nu_{ii} V_{Ti}^2 \omega_{0i}^{-2}$, where V_{Ti} is the ion thermal velocity and ν_{ii} is the ion–ion collision frequency. The damping rate of ion sound is $\nu_{\rm eff} \approx \eta k^2$, where k is the sound wavenumber. The number of ion sound harmonics, n, is determined by two conditions: (i) ion sound dispersion should be relatively small ($n^2q^2c_s^2 \ll \omega_{0i}^2$, where q is the wavenumber of the fundamental sound mode, $c_s = (\kappa T_e/M)^{1/2}$ is the ion sound velocity, M is the ion mass, and κ is the Boltzmann constant) and (ii) the damping rate ν_{eff} should be much smaller than the circular frequency of sound (i.e., the nth sound harmonic should be weakly damped). Analyzing conditions (i) and (ii), we find the number of harmonics n in the zebra pattern.

2. INITIAL EQUATIONS, CONDITIONS OF SYNCHRONISM, AND EQUATIONS FOR THE AMPLITUDES OF WAVES

Let us consider a weakly relativistic monoenergetic proton beam[1] propagating with the velocity $V_0 \sim c/3$ (where c is the speed of light in vacuum) in a highly nonisothermal quasineutral plasma. The plasma electrons in the wave field are assumed to have a Boltzmann distribution [19]. The beam ions interact with the plasma via the electric field $\mathbf{E}$ (here, $\mathbf{E}\|0x\|\mathbf{k}_j$, where $\mathbf{k}_j$ is the wave vector of the jth mode). Beam–plasma interaction can be described in terms of quasi-hydrodynamic equations [19, 21],

$$\begin{gathered} E = -\frac{\partial\varphi}{\partial x};\ \ \frac{\partial E}{\partial x} = 4\pi e(\bar{\rho}_e - \rho_s - \rho_i); \\ \frac{\partial V_i}{\partial t} + V_i\frac{\partial V_i}{\partial x} = -\frac{e}{M}E - \nu_{\text{eff}}V_i;\ \ \frac{\partial \rho_i}{\partial t} + \frac{\partial}{\partial x}(\rho_i V_i) = 0; \\ \frac{\partial \bar{V}_s}{\partial t} + \bar{V}_s\frac{\partial \bar{V}_s}{\partial x} = -\frac{e}{M_o\gamma}\left[E + c^{-2}\bar{V}_s^2E\right]; \\ \frac{\partial}{\partial t}(\gamma\bar{\rho}_s) + \frac{\partial}{\partial x}(\gamma\bar{\rho}_s\bar{V}_s) = 0, \end{gathered} \tag{1}$$

where e, M_0, and M are the electron charge, the rest mass of a beam ion, and the mass of a plasma ion, respectively; $\bar{\rho}_e = N_0\exp(e\varphi/\kappa T_e)$; φ is the electric potential; $\bar{\rho}_b = N_{0b} + \rho_b$, $\bar{V}_b = V_0 + V_b$; ρ_b, V_b, ρ_i, and V_i are deviations of the beam ion density, beam ion velocity, plasma ion density, and plasma ion velocity from their equilibrium values N_{0b}, V_0, N_0, and 0, respectively; and $\gamma = (1 - \bar{V}_b^2/c^2)^{-1/2}$.

Linearizing Eq. (1) with respect to perturbed quantities, which are assumed to vary in space and time as $\sim\exp(i\omega t - ikx)$, we obtain the dispersion relation for the beam–plasma system,

$$1 - \frac{\omega_{0i}^2}{\omega} - \frac{\omega_{0i}^2}{c_s^2k^2} - \frac{\omega_{0b}^2}{(\omega - kV_0)^2\left(1 - \frac{\omega - ck}{3ck}\right)} = 0, \tag{2}$$

where $\omega_{0b}^2 = 4\pi e^2N_{0b}M_0^{-1}$, $\omega_{0i}^2 = 4\pi e^2N_0M^{-1}$, ω is the circular frequency, and k is the wavenumber. For $V_0/c_s \gg 1$, $N_{0s}/N_0 \ll 1$, we obtain from Eq. (2) the following approximate dispersion relations:

$$\omega_1 \equiv \Omega \approx c_sk_1 \equiv c_smq, \tag{3}$$

$$\omega_{3,2} - k_{3,2}V_0 \approx \mp\omega_{0b} + \delta,\ \ \frac{\delta}{\omega_{0b}} \ll 1. \tag{4}$$

Dispersion relation (3) describes an ion sound wave with a positive energy, while dispersion relations (4) describe a slow (ω_3) and a fast (ω_2) beam wave having a negative and a positive energy, respectively. It is easy to see that the slow beam wave (ω_3, k_3), fast beam wave (ω_2, k_2), and sound wave (Ω, q) satisfy the synchronism conditions [22]. Taking into account Eqs. (3) and (4), we find from the synchronism conditions that

$$mq \approx 2\omega_{0s}V_0^{-1}. \tag{5}$$

Since ion sound dispersion is weak, the following cascade process is possible:

$$mq + mq \to 2mq + mq \to 3mq + mq\ldots mnq.$$

Let us expand the nonlinear terms in Eq. (1) in power series in perturbed quantities and retain the second-order terms. Then, applying a standard technique [14, 22], we obtain the following reduced equations for the complex amplitudes $a_j(\mu x, \mu t)$ ($j = 1, 2$) and b_k ($k = 1, 2, \ldots, mn$), where a_j are the amplitudes of the beam modes, b_k are the amplitudes of the ion sound modes, and μ is a small parameter (for simplicity, we consider a steady state, in which $\partial/\partial t = 0$):

$$\begin{gathered} V_0\frac{\partial a_1}{\partial x} = \sigma_1a_2b_m^*,\ \ V_0\frac{\partial a_2}{\partial x} = \sigma_2a_1b_m, \\ \sigma_1 \approx \sigma_2 = \sigma, \end{gathered} \tag{6}$$

$$\begin{gathered} c_s\frac{\partial b_m}{\partial x} = \beta a_1^*a_2 \\ + i\delta\left(b_{2m}b_m^* + b_{3m}b_{2m}^* + \ldots b_{nm}b_{(n-1)m}^*\right) - \nu b_m, \\ c_s\frac{\partial b_{2m}}{\partial x} = 2\left(b_m^*b_{3m} + b_{2m}^*b_{4m} + \ldots b_{nm}b_{(n-2)m}^* + b_m^2\right) - 4\nu b_m, \\ c_s\frac{\partial b_{3m}}{\partial x} = 3\left(b_{4m}b_m^* + b_{5m}b_{2m}^* + \ldots b_{nm}b_{(n-3)m}^* + b_{2m}b_m\right) - 9\nu b_m. \end{gathered} \tag{7}$$

Here, $\nu = (V_{Ti}/\nu_{ii})q$. The solution to Eq. (7) in a given field $|a_{1,2}| \gg |b_k|$ can be found in [12] (see also [23]). Analysis shows that σ_1, σ_2, and β have the same sign, i.e., Eqs. (6) and (7) describe a "stabilized explosion" [15].[2]

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[1] The beam can be considered monoenergetic under the condition [20] $(N_b/N_0)^{1/3}(V_0/V_{Tb})^2 \ll 1$, where N_b, V_{Tb}, and V_0 are the beam density, the beam thermal velocity, and the equilibrium velocity of beam particles, respectively.

[2] It should be noted that conventional beam instability can be ignored if its growth rate is much smaller than the inverse explosion time, i.e., if $(N_{0b}/N_0)^{1/3} \ll \omega_{0i}/(\sigma|a|)$. Moreover, beam instability is suppressed by a strong field [24].

3. QUALITATIVE ESTIMATES

Let us take the following parameters of the beam–plasma system as applied to the solar corona: $N_0 \sim 5 \times 10^9$ cm^{-3}, $N_b/N_0 \sim 10^{-3}$, $V_0 \le c/3 \sim 10^{10}$ cm/s, $T_e \sim 10^6$ K, and $T_i \sim 10^5$ K. In this case, we have $c_s \sim 10^7$ cm/s and $\nu_{ii} \sim (0.5–1)$ s^{-1}. The maximum number of sound harmonics can be estimated from conditions (i) and (ii) (see Introduction) and the conditions for radiation to escape from the solar corona, $\omega^t > \omega_0$ (which corresponds to the circular frequency of the scattered electromagnetic wave, $\omega^t \approx 634$ MHz). Then, we have $7 \times 10^2 \le mn \ll 15 \times 10^3$. The frequency $\omega_3 \sim \omega \gg \omega_{0i}$ is assumed to be given,[3] while the quantities ω_2, k_2, k_3, Ω, and q are determined from Eqs. (3) and (4) and the synchronism conditions. For a constant magnetic field of ~30 G, the growth rate of ion sound Γ_w in a given field of whistlers was estimated in [12] ($\Gamma_w \sim |a_w|^2\sigma_w$, where $|a_w|$ is the whistler amplitude and σ_w is the matrix interaction coefficient). In a given field of beam waves, $|a_b|$, the growth rate of ion sound is $\Gamma_b \sim |a_b|^2\beta$, where $\beta \approx 4 \times 10^9 em/(M_0c)$ and the quantity σ (see Eq. (6)) is $\sigma \sim 10^2 em/(M_0c)$. In this case, the amplitude of the fundamental ion sound harmonic increases as $b_1 \sim \Gamma_b x$. Estimates show that $\Gamma_b/\Gamma_w \sim 6 \times 10^5|a_b|^2/|a_w|^2$. Thus, the proposed mechanism of ion sound generation is much more efficient than that described in [12].

In the above estimates for β and σ, the wavelength and initial frequency of ion sound are assumed to be ~100 m and ~1.0 kHz, respectively (see Eqs. (3), (5)). The circular frequency of the slow beam mode is $\omega_3 \sim 7 \times 10^2\omega_{0i}$ (which corresponds to ~10 GHz).

The generated ion sound is scattered by fast protons moving with a velocity of $V \sim V_0 \sim 10^{10}$ cm/s. According to the mechanism described in [12],[4] the frequency of radiation emitted by the source is $\omega^t \approx mqnV$ and the frequency spacing between stripes is $\delta\omega^t = mqV$. Taking into account Eq. (5), for a radiation frequency of ≥634 MHz and the given parameters $N_0 \sim 5 \times 10^9$ cm^{-3} and $N_b/N_0 \sim 10^{-3}$, we find that $m = 15$. Hence, the frequency spacing between neighboring stripes is $\delta\omega^t \approx 15$ MHz. This value of $\delta\omega^t$ corresponds to the observed period of the zebra pattern in the decimeter wave band [4].

The proposed mechanism also provides the observed increase in $\delta\omega^t$ (for the same nth harmonics, the factor m increases with increasing frequency). Discrete radiation stripes can exist if the width of the stripes is smaller than the frequency spacing between them. This condition imposes restrictions on the velocity spread of the beam ions. According to estimates made in [12], the proton beam should be highly monoenergetic ($\Delta V_0/V_0 < 10^{-3}$ in our case).

The dips between the radiation strips of the zebra pattern can be associated with the disruption of loss-cone instability (which is responsible for continuum radiation) due to additional injection of fast particles into the loss cone. This mechanism was proposed in [25] to explain negative bursts.

4. CONCLUSIONS

The above mechanism based on the stabilization of explosive instability accompanying a cascade increase in the amplitudes of ion sound harmonics proves to be more efficient than the mechanism associated with the decay of whistlers into ion sound harmonics [12]. The proposed mechanism provides a greater number of zebra pattern harmonics with a frequency spacing that is independent of the ratio between the plasma and cyclotron frequencies in the source and increases with increasing frequency (in accordance with observations). In this case, no additional rigid constraints are imposed and there is only the conventional requirement related to the generation of monoenergetic particle beams, which usually occur in any strong burst.

In this context, it should be noted that each improvement of the existing models is usually accompanied by imposing new restrictions on the parameters of the plasma and fast particles in the source. For example, it was shown in [26] that the double plasma resonance provides a larger number of the zebra pattern harmonics only if fast particles possess a steep power-law energy spectrum. In the whistler model [4, 8], the number of stripes in the zebra pattern is large only if the magnetic trap is filled with whistler wave packets in the regime of wave-induced quasilinear diffusion of fast particles. New models of stripe formation in the zebra pattern by electromagnetic waves propagating through regular density inhomogeneities in the solar corona assume the presence of small-scale inhomogeneities with a spatial period of several meters. All these models and their capabilities of explaining new observational data are described in more detail in review [27].

It should be noted that the results obtained in the present study can also be applied to explain the generation of high-power low-frequency radiation in the atmospheres of stars and pulsars, estimate the efficiency of plasma heating in ICF devices (heating of plasma targets by particle beams), and create high-power generators (amplifiers) of low-frequency radiation in laboratory gas-discharge and solid-state plasmas.

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[3] For given parameters N_0, N_{0b}, V_0, and c_s, from Eq. (2) with allowance for Eqs. (3) and (5), we readily obtain $\omega/\omega_{0i} \sim 10^2 \times (7 + \delta/(3\omega_{0b}))$, where δ is the quantity entering into Eq. (4).

[4] Here, we mean stimulated wave scattering by particles (by analogy with Eq. (11) in [22]), in which case we have $\omega^t - \Omega = (k^t - q)V$.

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